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Effective analytical solutions versus numerical treatments of Chavy-Waddy-Kolokolnikov bacterial aggregates model in phototaxic

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Abstract One of the important problems arising in biology science is the bacteria motion that surrenders to some operators as light, heat…,etc. The Chavy-Waddy-Kolokolnikov equation (CWKE) is considered one of the famous models in biology branch that is very valuable in the modeling bacteria collective formation attracted to the light. Hereby, we will study this effective model to construct the soliton behaviors of this model. Hereby, we will choose two of the impressive semi-analytical methods, namely, the extended simple equation method (ESEM) and the (G'/G)-expansion method to extract the analytical solutions of this model and derive the soliton behaviors through the 2-kind and 3-kind graphs of these obtained solutions. The two suggested methods are two famous of the ansatz methods that surrender to the homogenous balance rule, used to construct the exact solution for nonlinear partial differential equation and examined before for many other nonlinear evolution equations and usually realize good results. In addition, we will derive the numerical solutions identical for achieved analytical solutions by using the differential transform method (DTM) which is one of the most, efficient numerical methods. To show the newly of our results, we will make comparison for the obtained soliton solutions behaviors with that previously realized by other authors.

1 Introduction

Bacteria must survive in environments that restrict their growth. So, a set of complex responses to external stimuli, such as moving toward or away from certain nutrients or light, have been developed by them to increase the availability of required resources, i.e., it comes up with a different of mechanisms to adapt and survive in their environments, such as migrating to regions with higher nutrient concentrations or better living conditions. Chemotaxis, the directed movement of motile bacteria in response to chemical gradients, is the well-studied of these phenomena. But in this work, we concentrate on the other form of organism adaptation called phototoxic which is a motion that is oriented respect to the direction of light. There are a series of papers based on mathematical modeling, where D. Levy et al. [1–6] who suggested models to describe how phototaxis bacteria behave based on basic features extracted from observations and experiments. Rotation is one of the internal variables involved with excitation in some models, and it has been found that the sensitivity to perform phototaxis decreases if the colony is not rotated, [7]. From the latest, (CWK) suggested a streamlined system of equations for the probability that the bacteria move and get a fourth-order nonlinear partial differential equation (NLPDE), only depending on one parameter that brings the probabilities of moving to either side, staying, or changing direction according to the sensing distance [8]. The reduced model is of crawling type for bacterial aggregation, like the Cahn–Hillard equation [9]. The CWK equation includes three terms in the right hand side one of them is nonlinear which has a unique parameter that gives the aggregate extent and its value defines the instability region for structure shaping, the other two terms are linear called a reverse diffusion term and a long-range effect term [10].

CWK refers to the design of the movement of phototoxic bacterial aggregates using the random forms introduced by Levy et al. and has the form [11]:

$$v_t = -v_{xx} - v_{xxxx} + \sigma \left(\frac{v_x v_{xx}}{v} \right)_x. \tag{1}$$

When the transport is toward zones the reverse diffusion occurs where the emphasis progression of concentration gradient is high. For instance, it is the case, in which the Cahn–Hilliard equation for the juncture separation process [9, 12]. The fourth-order term sometimes deals with long-range terms where the impact of distant neighbors on the emphasis at a determined point is given [13,

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14]. While the third term is the only term containing the nonlinearity and the only parameter of the given model,σ, which dominates in the size of the aggregate and takes the form

$$\sigma = \frac{\delta(2\gamma+1)(\gamma+1)^2}{\left(\delta\left[1+\gamma(\gamma^2+2\gamma+3)\right]-2\lambda\right)'}. \tag{2}$$

where λ is the rate jump of bacterium moves with keeping its direction and δ is the deflection rate to a new direction, while γ is the bacterium's recognizing radius. The steady solution of Eq. (1) is discussed in [1] which depends on decreasing the model order, then discussed the system orbits by introducing the transformation

$$v(x,t) = e^{w(x,t)}. \tag{3}$$

Using Eq. (3), then Eq. (1) surrenders to the form

$$\begin{aligned} w_t &= -w_x^2 - w_{xx} - w_{xxxx} + (4\sigma-6)w_x^2 w_{xx} \\ &+(\sigma-4)w_x w_{xxx} + (\sigma-3)w_{xx}^2 + (\sigma-1)w_x^4. \end{aligned} \tag{4}$$

For Eq. (4), four possible values of σ can be considered. For simplicity it is convenient to use the following variable change $\eta = x - \omega t$ which will transform Eq. (4) to

$$\begin{aligned} &\omega w_\eta - w_\eta^2 - w_{\eta\eta} - w_{\eta\eta\eta\eta} + (4\sigma-6)w_\eta^2 w_{\eta\eta} \\ &+(\sigma-4)w_\eta w_{\eta\eta\eta} + (\sigma-3)w_{\eta\eta}^2 + (\sigma-1)w_\eta^4 = 0. \end{aligned} \tag{5}$$

where ω is the wave velocity constancy, finally one can obtain a simple form for Eq. (5) with the help of the substitution $R = w_\eta$ as

$$\begin{aligned} &\omega R - R^2 - R_\eta - R_{\eta\eta\eta} + (4\sigma-6)R^2 R_\eta \\ &+(\sigma-4)R R_{\eta\eta} + (\sigma-3)R_\eta^2 + (\sigma-1)R^4 = 0. \end{aligned} \tag{6}$$

Recent studies to construct the soliton solutions of some models have been demonstrated see for example Kaplan and Akbulut, [15] who employed the modified simple equation method and the exp($-\varphi(\xi)$)method to find the exact solutions with various forms including hyperbolic, trigonometric, rational and exponential functions of fractional differential equations systems with conformable fractional derivative to the approximate long water wave equation, Wang, et al. [16] who applied auxiliary equation method for obtaining new traveling wave solutions for two interesting proposed systems, namely the Kaup-Boussinesq system and generalized Hirota–Satsuma coupled KdV system with beta time fractional derivative, Arnous, et al. [17] who applied the enhanced Kudryashov's and improved extended tanh-function techniques to find the optical solutions in forms of bright soliton, singular soliton, singular periodic wave, Jacobi elliptic function, exponential function, Weierstrass function, and dark soliton solutions for the perturbed Chen–Lee–Liu equation with Kudryashov's arbitrary refractive index model, Akbulut and Taşcan [18], who studied the fractional order to the modified Korteweg–de Vries equation and reduced fractional order of this model to fractional order ordinary differential equation with Erdelyi–Kober fractional differential operator to find solutions for this equation.

The remained of this article is prepared as follow: In Sect. (3), the analytical solution of Eq. (6) using ESEM [19–22] will be introduced. Also, the analytical solution of Eq. (6) using (G'/G)-expansion method [23, 24] will be discussed in Sect. (4). While the numerical treatment corresponding to the two mentioned analytical method using DTM [25–29] is introduced in Sect. (5). Finally, the conclusion and the references are given at the end of this work.

2 The ESEM structure

To introduce ESEM solution forms, let us introduce the general structure of any NLPDE

$$\mathrm{T}(R, R_x, R_t, R_{xx}, R_{tt}, \ldots) = 0. \tag{7}$$

When we apply the transformation $R(x,t) = R(\eta), \quad \eta = x - \omega t$ on Eq. (7) it becomes

$$\Omega(R, R', R'', \ldots) = 0. \tag{8}$$

where T in terms of $R(x,t)$ and its partial differentiation, while Ω in terms of $u(\eta)$ and its total derivatives.

The ESEM introduces the solution of Eq. (8) to be

$$R(\eta) = \sum_{i=-M}^{M} A_i \phi^i(\eta). \tag{9}$$

where A_i are parameters to be determined, M is the balance number and $\phi(\eta)$ will be explored from

$$\phi'(\eta) = a_0 + a_1\phi + a_2\phi^2 + a_3\phi^3. \tag{10}$$

Then a_0, a_1 and a_2 are arbitrary constants to be calculated.

(1) Firstly if $a_1 = a_3 = 0$ Eq. (10) will be Riccati equation whose solutions are

$$\phi(\eta) = \frac{\sqrt{a_0 a_2}}{a_2} \tan\left[\sqrt{a_0 a_2}(\eta + \eta_0)\right],\ a_0 a_2 \succ 0. \tag{11}$$

$$\phi(\eta) = \frac{\sqrt{-a_0 a_2}}{a_2} \tanh\left[\sqrt{-a_0 a_2}\left(\eta - \frac{\rho_1 \ln \eta_0}{2}\right)\right],\ a_0 a_2 \prec 0,\ \rho_1 = \pm 1, \eta \succ 0. \tag{12}$$

(2) Secondly, if $a_0 = a_3 = 0$, Eq. (10) will be Bernoulli equation whose solutions are

$$\phi(\eta) = \frac{a_1 Exp[a_1(\eta + \eta_0)]}{1 - a_2 Exp[a_1(\eta + \eta_0)]}, a_1 \succ 0. \tag{13}$$

$$\phi(\eta) = \frac{-a_1 Exp[a_1(\eta + \eta_0)]}{1 + a_2 Exp[a_1(\eta + \eta_0)]}, a_1 \prec 0. \tag{14}$$

Moreover, Eq. (10) has general solution in the form

$$\phi(\eta) = \frac{-1}{a_2}\left(a_1 - \sqrt{4a_1 a_2 - a_1^2} \tan\left[\frac{\sqrt{4a_1 a_2 - a_1^2}}{2}(\eta + \eta_0)\right]\right) 4a_1 a_2 \succ a_1^2\ , a_2 \succ 0. \tag{15}$$

$$\phi(\eta) = \frac{1}{a_2}\left(a_1 + \sqrt{4a_1 a_2 - a_1^2} \tanh\left[\frac{\sqrt{4a_1 a_2 - a_1^2}}{2}(\eta + \eta_0)\right]\right) 4a_1 a_2 \succ a_1^2\ , a_2 \prec 0. \tag{16}$$

where η_0 is the integration constant.

Let us apply this technique on Eq. (6) for which the balance number is $M = 1$ hence the solution is:

$$R(\eta) = \sum_{i=-1}^{1} A_i \phi^i(\eta) = \frac{A_{-1}}{\phi} + A_0 + A_1 \phi. \tag{17}$$

Firstly: The first family in which $a_1 = a_3 = 0 \Rightarrow \phi' = a_0 + a_2 \phi^2$, from (17) we have

$$R' = \frac{-a_0 A_{-1}}{\phi^2} + (A_1 a_0 - a_2 A_{-1}) + a_2 A_1 \phi^2. \tag{18}$$

$$R'' = \frac{2a_0^2 A_{-1}}{\phi^3} + \frac{2a_0 a_2 A_{-1}}{\phi} + 2a_0 a_2 A_1 \phi + 2a_2^2 A_1 \phi^3. \tag{19}$$

$$R''' = -\frac{6a_0^3 A_{-1}}{\varphi^4} - \frac{8a_0^2 a_2 A_{-1}}{\varphi^2} - 2a_0 a_2^2 A_{-1} + 2a_0^2 a_2 A_1 + 8\varphi^2 a_0 a_2^2 A_1 + 6\varphi^4 a_2^3 A_1. \tag{20}$$

Using the relations (17–20) into Eq. (6), set the various orders of φ^i equal zero, a generated system of equations will be detected from which these results are explored

$$(1)\ \ \omega \to 2\left(A_0 + 4A_0^3\right), \sigma \to 0, a_0 \to \frac{A_0^2}{A_1}, a_2 \to -A_1, A_{-1} \to 0. \tag{21}$$

$$(2)\ \ \omega \to -2\left(A_0 + 4A_0^3\right), \sigma \to 0, a_0 \to -\frac{1 + 3A_0^2}{A_1}, a_2 \to -A_1, A_{-1} \to 0. \tag{22}$$

$$(3)\ \ \omega \to 2\left(A_0 + 4A_0^3\right), \sigma \to 0, A_1 \to 0, a_0 \to A_{-1}, a_2 \to -\frac{A_0^2}{A_{-1}}. \tag{23}$$

$$(4)\ \ \omega \to -2\left(A_0 + 4A_0^3\right), \sigma \to 0, A_1 \to 0, a_0 \to A_{-1}, a_2 \to \frac{1 + 3A_0^2}{A_{-1}}. \tag{24}$$

$$(5)\ \ \omega \to 2\left(A_0 + 4A_0^3\right), \sigma \to 0, A_1 \to \frac{A_0^2}{4A_{-1}}, a_0 \to A_{-1}, a_2 \to -\frac{A_0^2}{4A_{-1}}. \tag{25}$$

$$(6)\ \ \omega \to -2\left(A_0 + 4A_0^3\right), \sigma \to 0, A_1 \to -\frac{1 + 3A_0^2}{4A_{-1}}, a_0 \to A_{-1}, a_2 \to \frac{1 + 3A_0^2}{4A_{-1}}. \tag{26}$$

For the first result Eq. (21) with $A_0,\ A_1 = 1,$ we have

$$\omega \to 10,\ \sigma \to 0, a_0 \to 1, a_2 \to -1, A_1 = A_0 = 1, A_{-1} \to 0, a_0 a_2 \prec 0. \tag{27}$$

Then according to the first family of the ESEM Eq. (12) the solution is:

$$\phi(\eta) = -\tanh[\eta + 0.3],\ \rho_1 = -1, \eta_0 = 2. \tag{28}$$

Hence, the solution form of Eq. (6) is:

$$R(\eta) = 1 - \tanh[\eta + 0.3]. \tag{29}$$

Since $R = w_\eta$ then

$$w = \int R(\eta)d\eta = \eta - \ln(\cosh[\eta + 0.3]). \tag{30}$$

With taking the integration equal zero. Also $v(x, t) = e^{w(x,t)}$. then the solution of our model Eq. (1) is:

$$v(x,t) = e^{(x-10t)}\ \mathrm{sech}(x - 10t + 0.3). \tag{31}$$

By the same procedure for the second result Eq. (22)

$$\omega \to -10, \sigma \to 0, a_0 \to -4, a_2 \to -1, A_1 = A_0 = 1, A_{-1} \to 0, a_0 a_2 \succ 0. \tag{32}$$

The first family of the ESEM Eq. (11) implies

$$\phi(\eta) = -2\tan[2(\eta + \eta_0)]. \tag{33}$$

$$R(\eta) = 1 - 2\tan[2(\eta + \eta_0)]. \tag{34}$$

$$w = \int R(\eta)d\eta = \eta + \ln(\cos 2[\eta + 2]), \eta_0 = 2. \tag{35}$$

So, the other form solution of our model Eq. (1) becomes

$$v(x,t) = e^{(x+10t)}\ \cos(2x + 20t + 4). \tag{36}$$

Secondly: The second family in which $a_0 = a_3 = 0 \Rightarrow \phi' = a_1\phi + a_2\phi^2$,

$$R' = \frac{-a_1 A_{-1}}{\varphi} - a_2 A_{-1} + A_1 a_1 \varphi + a_2 A_1 \varphi^2. \tag{37}$$

$$R'' = \frac{a_1^2 A_{-1}}{\varphi} + a_1 a_2 A_{-1} + a_1^2 A_1 \varphi + 3a_1 a_2 A_1 \varphi^2 + 2a_2^2 A_1 \varphi^3. \tag{38}$$

$$R''' = -\frac{a_1^3 A_{-1}}{\varphi} - a_1^2 a_2 A_{-1} + \varphi a_1^3 A_1 + 7\varphi^2 a_1^2 a_2 A_1 + 12\varphi^3 a_1 a_2^2 A_1 + 6\varphi^4 a_2^3 A_1. \tag{39}$$

Putting the relations (17) and (37–39) into Eq. (6), setting the coefficients of various orders of φ^i equal to zero, a generated system of equations will be detected from which these results emerge

$$(1)\ \ \omega \to A_0 + A_0^3, \sigma \to 0, a_1 \to -A_0, a_2 \to -A_1, A_{-1} \to 0. \tag{40}$$

$$(2)\ \ \omega \to A_0 + A_0^3, \sigma \to 0, a_1 \to \frac{1}{2}\left(-3A_0 - \sqrt{-4 - 3A_0^2}\right), a_2 \to -A_1, A_{-1} \to 0. \tag{41}$$

$$(3)\ \ \omega \to A_0 + A_0^3, \sigma \to 0, a_1 \to \frac{1}{2}\left(-3A_0 + \sqrt{-4 - 3A_0^2}\right), a_2 \to -A_1, A_{-1} \to 0. \tag{42}$$

$$(4)\ \omega \to a_1 + a_1^3, \sigma \to 0, A_0 \to 0, a_2 \to -A_1, A_{-1} \to 0. \tag{43}$$

Putting $A_0 = 1,\ A_1 = -1$. the first result Eq. (40) gives

$$\omega = 2, \sigma = 0, a_2 = 1, A_{-1} = 0, a_1 = -1 \prec 0. \tag{44}$$

From Eq. (14) we have $\phi(\eta) = \frac{Exp[-(\eta+\eta_0)]}{1+Exp[-(\eta+\eta_0)]}$.

From which we get

$$R(\eta) = 1 - \frac{Exp[-(\eta + \eta_0)]}{1 + Exp[-(\eta + \eta_0)]}. \tag{45}$$

$$w(\eta) = \int R(\eta)d\eta = \ln(1 + Exp[\eta + 2]), \eta_0 = 2. \tag{46}$$

So, the other form solution of our model Eq. (1) becomes

$$v(x,t) = e^{w(x,t)} = 1 + e^{(x-2t+2)}. \tag{47}$$

Also for the last result Eq. (43), by the same algorithm we get another equivalent solution for the proposed model Eq. (1) as

$$A_1 = -1, a_1 = 1, \omega = 2, \sigma = 0, a_2 = 1, A_{-1} = A_0 = 0, a_1 = 1 \succ 0. \tag{48}$$

$$R(\eta) = \frac{Exp[\eta + \eta_0]}{1 - Exp[\eta + \eta_0]}. \tag{49}$$

$$w(\eta) = \int R(\eta)d\eta = -\ln(1 - Exp[\eta + 2]), \eta_0 = 2. \tag{50}$$

So, the other form solution of our model Eq. (1) becomes

$$v(x,t) = e^{w(x,t)} = \frac{1}{1 - e^{(x-2t+2)}}. \tag{51}$$

3 The algorithm of the (G'/G)-expansion method

According to this method the solution of Eq. (8) is:

$$R(\eta) = A_0 + \sum_{k=1}^{M} A_k \left[\frac{G'}{G}\right]^k, A_M \neq 0. \tag{52}$$

where the function $G(\eta)$ surrenders to the auxiliary equation $G'' + \mu G' + \lambda G = 0$ while the parameter M denotes to the balance number, Eq. (52) has the following forms of solutions.

(I) If $\mu^2 - 4\lambda \succ 0$ the solution is:

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\mu^2 - 4\lambda}}{2}\left(\frac{l_1 \sinh\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\eta + l_2 \cosh\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\eta}{l_1 \cosh\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\eta + l_2 \sinh\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\eta}\right) - \frac{\mu}{2}. \tag{53}$$

(II) If $\mu^2 - 4\lambda \prec 0$ then:

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\mu^2 - 4\lambda}}{2}\left(\frac{-l_1 \sin\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\eta + l_2 \cos\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\eta}{l_1 \cos\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\eta + l_2 \sin\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\eta}\right) - \frac{\mu}{2}. \tag{54}$$

(III) If $\mu^2 - 4\lambda = 0$:

$$\left(\frac{G'}{G}\right) = \left(\frac{l_2}{l_1 + l_2\eta}\right) - \frac{\mu}{2}. \tag{55}$$

where $l_1,\ l_2$ are constants.

Applying this method on the main problem Eq. (6) that has the balance $M = 1,$ hence the solution is:

$$R(\eta) = A_0 + A_1\left(\frac{G'}{G}\right). \tag{56}$$

$$R' = A_1\left(-\mu\left(\frac{G'}{G}\right) - \lambda - \left(\frac{G'}{G}\right)^2\right). \tag{57}$$

$$R'' = A_1\left(\left(\mu^2 + 2\lambda\right)\left(\frac{G'}{G}\right) + \mu\lambda + 3\mu\left(\frac{G'}{G}\right)^2 + 2\left(\frac{G'}{G}\right)^3\right). \tag{58}$$

$$R''' = A_1\left(-2\lambda^2 - \lambda\mu^2 - \left(8\lambda\mu + \mu^3\right)\left(\frac{G'}{G}\right) - \left(8\lambda + 7\mu^2\right)\left(\frac{G'}{G}\right)^2 - 12\mu\left(\frac{G'}{G}\right)^3 - 6\left(\frac{G'}{G}\right)^4\right). \tag{59}$$

When the relations (56–59) are emerged into Eq. (6), setting the coefficients of various orders of $\left(\frac{G'}{G}\right)^i$ equal to zero, a system of equations will be explored and by solving it we get

$$(1)\ \omega \to -\mu\left(1 + \mu^2\right) + \left(2 + 6\mu^2\right)A_0 - 12\mu A_0^2 + 8A_0^3, \lambda \to (\mu - A_0)A_0, A_1 \to 1, \sigma \to 0. \tag{60}$$

$$(2)\ \omega \to \mu + \mu^3 - 2\left(1 + 3\mu^2\right)A_0 + 12\mu A_0^2 - 8A_0^3, \lambda \to 1 + \mu^2 - 3\mu A_0 + 3A_0^2, A_1 \to 1, \sigma \to 0. \tag{61}$$

4 (1) For the first result, this can be reduced to

$$\omega = 10, \sigma = 0, A_0 = A_1 = 1, \ = 0, \mu = 0, \lambda = -1. \tag{62}$$

According to this result, $\mu^2 - 4\lambda = 4 \succ 0$ that achieves the $\left(\frac{G'}{G}\right)$ form (I) which is:

$$\left(\frac{G'}{G}\right) = \frac{\sinh \eta + 2\cosh \eta}{\cosh \eta + 2\sinh \eta}, \ l_1 = 1, \ l_2 = 2.$$

$$R(\eta) = 1 + \frac{\sinh \eta + 2\cosh \eta}{\cosh \eta + 2\sinh \eta}. \tag{63}$$

$$w(\eta) = \int R(\eta) d\eta = \eta + \ln(\cosh \eta + 2\sinh \eta). \tag{64}$$

Hence, the solution of Eq. (1) is:

$$v(x,t) = e^{w(x,t)} = e^{x-10t}[\cos 2i(x - 10t) + 2\sin 2i(x - 10t)]. \tag{65}$$

5 (2) For the second result, this can surrender to

$$\omega = -10, \sigma = 0, A_0 = A_1 = 1, \ = 0, \mu = 0, \lambda = 4. \tag{66}$$

From these values we have $\mu^2 - 4\lambda = -16 \prec 0$ then the solution is:

$$\left(\frac{G'}{G}\right) = 2i\left(\frac{-\sin 2i\eta + 2\cos 2i\eta}{\cos 2i\eta + 2\sin 2i\eta}\right), \ l_1 = 1, \ l_2 = 2.$$

$$R(\eta) = 1 + 2i\left(\frac{-\sin 2i\eta + 2\cos 2i\eta}{\cos 2i\eta + 2\sin 2i\eta}\right). \tag{67}$$

$$w(\eta) = \int R(\eta) d\eta = \eta + \ln(\cos(2i\eta) + 2\sin(2i\eta)). \tag{68}$$

Consequently, the solution of Eq. (1) is:

$$v(x,t) = e^{w(x,t)} = e^{x+10t}[\cos 2i(x + 10t) + 2\sin 2i(x + 10t)]. \tag{69}$$

6 The DTM algorithm

In many nonlinear sciences fields that can be described by the NLSE it is very difficult to achieve the exact solutions, for this reasons we utilize the numerical methods to construct the numerical solutions. Some of these problems have exact solutions and it is more convenient to construct the corresponding numerical solutions. The DTM is considered one of these numerical methods that we will use for this purpose. The k$^{\text{th}}$ derivative of function $v(x)$ according to the DTM can be written as

$$V(k) = \frac{1}{k!}\left[\frac{d^k v(x)}{dx^k}\right]_{x=0}. \tag{70}$$

where $v(x)$ denotes to the base function and $V(k)$ denotes to the converted functions whose reverse is

$$v(k) = \sum_{k=0}^{\infty} V(k)x^k \cong \sum_{k=0}^{N} \frac{x^k}{k!}\left[\frac{d^k v(x)}{dx^k}\right]_{x=0}. \tag{71}$$

Through Eq. (70), Eq. (71) the DTM is defined by Taylor expansion, while previously these two relations are defined at $x = 0$, for more information's, main theorems and basic rules see [20–23].

Let us now introduce the numerical solutions corresponding to the exact solutions explored by the ESEM and (G'/G) using DTM.

6.1 Firstly for the solutions obtained via ESEM

According to the first result of the first family in ESEM Eq. (6) with $\sigma = 0$, $\omega = 10$ becomes

$$10R - R^2 - R_\eta - R_{\eta\eta\eta} - 6R^2 R_\eta - 4RR_{\eta\eta} - 3R_\eta^2 - R^4 = 0. \tag{72}$$

Applying DTM on Eq. (72) whose initial conditions derived from Eq. (29) will be

$$R(0) = 0.708687, R_\eta(0) = -0.915, R_{\eta\eta}(0) = 0.5333. \tag{73}$$

We get

$$\begin{aligned}
&10T(k)-\sum_{s=0}^{k}T(s)T(k-s)-(k+1)T(k+1)-(k+1)(k+2)(k+3)T(k+3)\\
&-6\sum_{s=0}^{k}\sum_{m=0}^{s}(m+1)T(m+1)T(s-m)T(k-s)-4\sum_{r=0}^{k}(s+1)(s+2)T(s+2)T(k-s)\\
&-3\sum_{s=0}^{k}(s+1)(k-s+1)T(s+1)T(k-s+1)-F(k)=0.
\end{aligned} \tag{74}$$

where $T(k)$ the transformed function of $R(\eta)$ while $F(k)$ the transformation of R^4 given by

$$F(k)=\begin{cases}(T(0))^4; \quad k=0.\\ \dfrac{1}{T(0)}\displaystyle\sum_{s=1}^{k}\frac{5s-k}{k}T(s)F(k-s); \quad k\geq 1.\end{cases} \tag{75}$$

For which these recurrence relations will be detected

$$T(k+3)=\frac{1}{(k+1)(k+2)(k+3)}\left(\begin{array}{l}10T(k)-\displaystyle\sum_{s=0}^{k}T(s)T(k-s)-(k+1)T(k+1)\\ -6\displaystyle\sum_{s=0}^{k}\sum_{m=0}^{s}(m+1)T(m+1)T(s-m)T(k-s)\\ -4\displaystyle\sum_{s=0}^{k}(s+1)(s+2)T(s+2)T(k-s)\\ -3\displaystyle\sum_{r=0}^{k}(s+1)(k-s+1)T(s+1)T(k-s+1)-F(k)\end{array}\right). \tag{76}$$

where $\{k=0,1,2,3,...\}$.

From (76) with the help of DTM algorithm the solution of (72) is:

$$\begin{aligned}R(\eta)\cong\ &0.708687-0.915\eta+0.26659\eta^2+0.996894\eta^3\\ &-1.16207\eta^4-1.22987\eta^5+0.916\eta^6+0.3576\eta^7-0.177\eta^8+....\end{aligned} \tag{77}$$

$$\begin{aligned}w(\eta)=\int R(\eta)d\eta=\ &0.708687\eta-0.457568\eta^2+0.08886\eta^3+0.24922\eta^4\\ &-0.2324\eta^5-0.2049785\eta^6+0.1309\eta^7+0.0447\eta^8-0.01969\eta^9+....\end{aligned} \tag{78}$$

Hence the DTM solution of the suggested model Eq. (1) is:

$$v(x,t)=e^{w(x,t)}=e^{(x-10t)\left[\begin{array}{l}0.708687-0.4575(x-10t)+0.08886(x-10t)^2+0.2492(x-10t)^3\\ -0.2324(x-10t)^4-0.20497(x-10t)^5+0.13(x-10t)^6+0.0447(x-10t)^7-0.01969(x-10t)^8\end{array}\right]}. \tag{79}$$

This is compatible with the exact solution Eq. (31)

Also, by the same procedure for the second result of the first family of the ESEM we can extract the DTM solution corresponding to the exact solution Eq. (36)

$$\begin{aligned}R(\eta)_{DTM}\cong\ &-1.31564-9.3622\eta-21.6795\eta^2-62.68493\eta^3-174.0619\eta^4\\ &-489.973\eta^5-1374.3951\eta^6-3859\eta^7-10834\eta^8+....\end{aligned} \tag{80}$$

This is very close to the series expansion of the corresponding analytical solution Eq. (34)

$$\begin{aligned}R(\eta)_{Exact}\cong\ &-1.31564-9.3622\eta-21.6795\eta^2-62.68493\eta^3-174.0619\eta^4\\ &-489.973\eta^5-1374.3951\eta^6-3859\eta^7-10834\eta^8+....\end{aligned} \tag{81}$$

$$\begin{aligned}w(\eta)=\int R(\eta)d\eta=\ &-1.31564\eta-4.6811\eta^2-7.2265\eta^3-15.6712325\eta^4\\ &-34.81\eta^5-81.6623\eta^6-196.342\eta^7-482.43\eta^8-1203\eta^9+....\end{aligned} \tag{82}$$

Then the solution of the proposed model Eq. (1) is:

$$v(x,t) = \mathrm{e}^{(x+10t)\begin{pmatrix}0.708687-0.4575684(x+10t)+0.08886(x+10t)^2+0.24922(x+10t)^3\\-0.2324(x+10t)^4-0.20(x+10t)^5+0.13(x+10t)^6+0.0447(x+10t)^7-0.01969(x+10t)^8\end{pmatrix}}. \tag{83}$$

Also, we can apply the DTM for the second family of the ESEM by considering two results achieved by the ESEM, for the first result with $\sigma = 0,\ \omega = 2$ Eq. (6) becomes

$$2R - R^2 - R_\eta - R_{\eta\eta\eta} - 6R^2 R_\eta - 4R R_{\eta\eta} - 3R_\eta^2 - R^4 = 0. \tag{84}$$

Using DTM algorithm Eq. (84) with initial conditions obtained from Eq. (45)

$$R(0) = 0.880797,\ R_\eta(0) = 0.1049935,\ R_{\eta\eta}(0) = -0.0799625. \tag{85}$$

We get

$$\begin{aligned}&2T(k) - \sum_{s=0}^{k} T(s)T(k-s) - (k+1)T(k+1) - (k+1)(k+2)(k+3)T(k+3)\\&-6\sum_{s=0}^{k}\sum_{m=0}^{s}(m+1)T(m+1)T(s-m)T(k-s) - 4\sum_{s=0}^{k}(s+1)(s+2)T(s+2)T(k-s)\\&-3\sum_{s=0}^{k}(s+1)(k-s+1)T(s+1)T(k-s+1) - F(k) = 0.\end{aligned} \tag{86}$$

From which one can obtain the following recurrence relation

$$T(k+3) = \frac{1}{(k+1)(k+2)(k+3)}\begin{pmatrix}2T(k) - \sum_{s=0}^{k} T(s)T(k-s) - (k+1)T(k+1)\\-6\sum_{s=0}^{k}\sum_{m=0}^{s}(m+1)T(m+1)T(s-m)T(k-s)\\-4\sum_{s=0}^{k}(s+1)(s+2)T(s+2)T(k-r)s\\-3\sum_{s=0}^{k}(s+1)(k-s+1)T(s+1)T(k-s+1) - F(k)\end{pmatrix}. \tag{87}$$

where $\{k = 0,\ 1,\ 2,\ 3,\ \ldots\}$.

From (87) with the help of DTM algorithm the solution of (84) is:

$$\begin{aligned}R(\eta)_{DTM} \cong\ &0.880797 + 0.10499\eta - 0.03998\eta^2 + 0.006475\eta^3 + 0.000866\eta^4\\&- 0.00072355\eta^5 + 0.00014783\eta^6 + 0.000009524\eta^7 - 0.000013421\eta^8 + \ldots.\end{aligned} \tag{88}$$

This is very close to the series expansion of the corresponding analytical solution Eq. (45)

$$\begin{aligned}R(\eta)_{Exact} \cong\ &0.880797 + 0.10499\eta - 0.03998\eta^2 + 0.006475\eta^3 + 0.000866\eta^4\\&- 0.00072355\eta^5 + 0.00014783\eta^6 + 0.000009524\eta^7 - 0.000013421\eta^8 + \ldots.\end{aligned} \tag{89}$$

From (88)

$$\begin{aligned}w(\eta)_{DTM} = \int R(\eta)d\eta = &\ 0.880797\eta + 0.052495\eta^2 - 0.0133\eta^3 + 0.0016\eta^4 + 0.0001732\eta^5\\&- 0.00012\eta^6 + 0.00002\eta^7 + 0.00000119\eta^8 - 0.00000149\eta^9 + \ldots.\end{aligned} \tag{90}$$

Then the solution of the proposed model Eq. (1) is

$$v(x,t)_{DTM} = e^{(x-2t)\begin{pmatrix}0.880797+0.052495(x-2t)-0.0133(x-2t)^2+0.0016(x-2t)^3+0.00017(x-2t)^4\\-0.00012(x-2t)^5+0.00002(x-2t)^6+0.00000119(x-2t)^7-0.00000149(x-2t)^8\end{pmatrix}}. \tag{91}$$

By the same way, for the last result of the second family in ESEM with $\omega = 2,\ \sigma = 0$.

We get the same Eq. (84) with initial conditions obtained from Eq. (49)

$$R(0) = -1.15652,\ R_\eta(0) = 0.1812,\ R_{\eta\eta}(0) = -0.23768. \tag{92}$$

Applying the DTM on Eq. (84) with the help of initial conditions (92) we get the same recurrence relation Eq. (87), taking different values of the parameter $k = 0, 1, 2, 3, \ldots$ we get

$$R(\eta)_{DTM} \cong -1.15652 + 0.181\eta - 0.1188398\eta^2 + 1.3785\eta^3 - 1.3888\eta^4 + 0.534\eta^5 - 0.16458584\eta^6 + 0.5583\eta^7 - 0.893695\eta^8 + \ldots. \tag{93}$$

This is very close to the series expansion of the corresponding analytical solution Eq. (49)

$$w(\eta)_{DTM} = \int R(\eta)d\eta = -1.15652\eta + 0.0905\eta^2 - 0.0396\eta^3 + 0.34463\eta^4 - 0.277765\eta^5 + 0.089\eta^6 - 0.0235\eta^7 + 0.069786\eta^8 - 0.099299\eta^9 + \ldots. \tag{94}$$

Then the solution of the proposed model Eq. (1) is:

$$v(x,t)_{DTM} = \mathrm{e}^{(x-2t)\begin{pmatrix}-1.15652+0.0905(x-2t)-0.0396\,(x-2t)^2-0.3446(x-2t)^3-0.277765(x-2t)^4\\ -0.089(x-2t)^5-0.0235(x-2t)^6-0.069786(x-2t)^7-0.099299(x-2t)^8\end{pmatrix}}. \tag{95}$$

6.2 Secondly for the solutions obtained via the (G'/G)-expansion

According to (G'/G)-expansion method two results are discussed, for the first result with $\sigma = 0$, $\omega = 10$ Eq. (6) becomes

$$10R - R^2 - R_\eta - R_{\eta\eta\eta} - 6R^2R_\eta - 4RR_{\eta\eta} - 3R_\eta^2 - R^4 = 0. \tag{96}$$

Using DTM algorithm Eq. (96) whose initial conditions explored from Eq. (63)

$$R(0) = 3, R_\eta(0) = -3, R_{\eta\eta}(0) = 12. \tag{97}$$

We get

$$10T(k) - \sum_{s=0}^{k} T(s)T(k-s) - (k+1)T(k+1) - (k+1)(k+2)(k+3)T(k+3) - 6\sum_{s=0}^{k}\sum_{m=0}^{s}(m+1)T(m+1)T(s-m)T(k-s) - 4\sum_{s=0}^{k}(s+1)(s+2)T(s+2)T(k-s) - 3\sum_{s=0}^{k}(s+1)(k-s+1)T(s+1)T(k-s+1) - F(k) = 0. \tag{98}$$

From which one can obtain the following recurrence relation

$$T(k+3) = \frac{1}{(k+1)(k+2)(k+3)}\begin{pmatrix} 10T(k) - \sum_{s=0}^{k} T(s)T(k-s) - (k+1)T(k+1) \\ -6\sum_{s=0}^{k}\sum_{m=0}^{s}(m+1)T(m+1)T(s-m)T(k-s) \\ -4\sum_{s=0}^{k}(s+1)(s+2)T(s+2)T(k-s) \\ -3\sum_{s=0}^{k}(s+1)(k-s+1)T(s+1)T(k-s+1) - F(k) \end{pmatrix}. \tag{99}$$

where $\{k = 0, 1, 2, 3, \ldots\}$.

From (99) with the help of DTM algorithm the solution of (96) is:

$$R(\eta)_{DTM} \cong 3 - 3\eta + 6\eta^2 - 11\eta^3 + 20\eta^4 - 36.4\eta^5 + 66.27\eta^6 - 120.638\eta^7 + 219.62\eta^8 + \ldots. \tag{100}$$

This is very close to the series expansion of the corresponding analytical solution Eq. (63)

$$R(\eta)_{Exact} \cong 3 - 3\eta + 6\eta^2 - 11\eta^3 + 20\eta^4 - 36.4\eta^5 + 66.27\eta^6 - 120.638\eta^7 + 219.62\eta^8 + \ldots. \tag{101}$$

From (100)

$$w(\eta)_{DTM} = \int R(\eta)d\eta = 3\eta - 1.5\eta^2 + 2\eta^3 - 2.75\eta^4 + 4\eta^5 - 6\eta^6 + 9.47\eta^7 - 15.1\eta^8 + 24.4\eta^9. \tag{102}$$

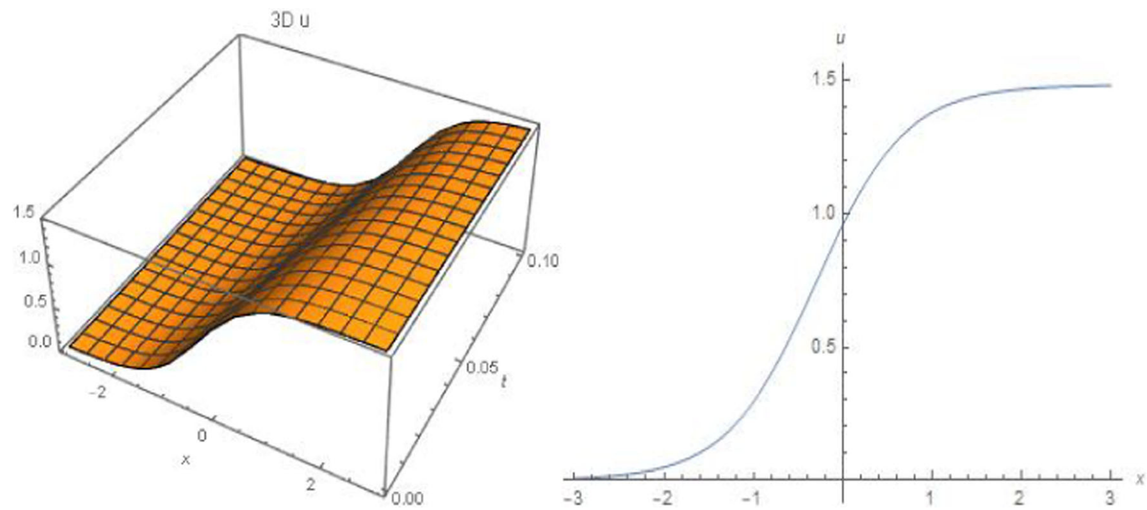

Fig. 1 The 2D & 3D graphs of the solution Eq. (31) when $\omega \to 10,\ \sigma \to 0,\ a_0 \to 1,\ a_2 \to -1,\ A_1 = A_0 = 1,\ A_{-1} \to 0,\ a_0a_2 \prec 0$.

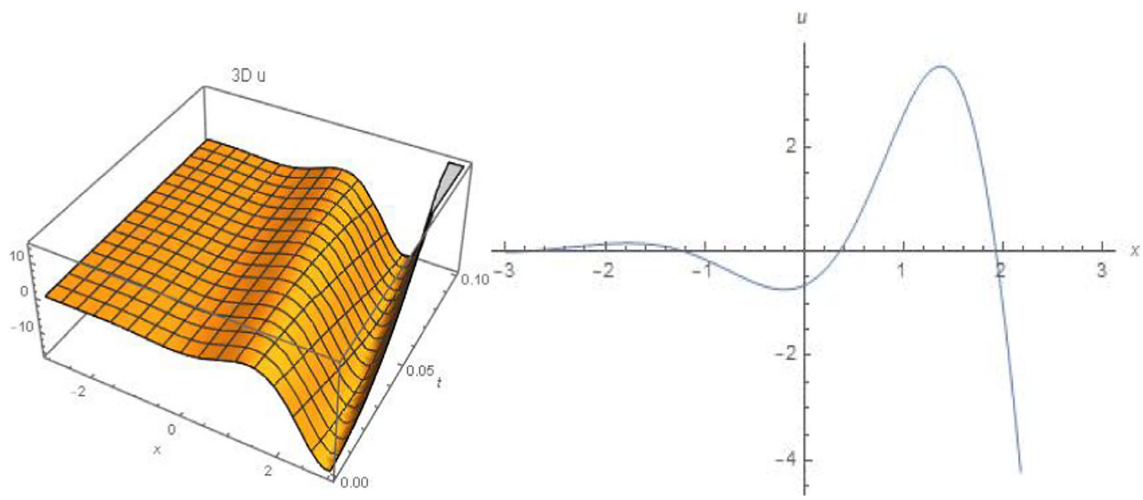

Fig. 2 The 2D & 3D graphs of the solution Eq. (36) when: $\omega \to -10,\ \sigma \to 0,\ a_0 \to -4,\ a_2 \to -1,\ A_1 = A_0 = 1,\ A_{-1} \to 0,\ a_0a_2 \succ 0$.

Then the solution of the proposed model Eq. (1) is:

$$v(x,t)_{DTM} = e^{(x-10t)\left(3-1.5\,(x-10t)+2(x-10t)^2-2.75(x-10t)^3+4(x-10t)^4-6(x-10t)^5+9.47(x-10t)^6-15(x-10t)^7+24.4(x-10t)^8\right)}. \tag{103}$$

7 Conclusion

Hereby of two interesting reliable methods, namely the ESEM and (G’/G)-method, the new optical soliton solutions of the CWK Model of Bacterial Aggregates will be explored. The new divers of optical soliton solutions achieved by the ESEM appear in forms of hyperbolic functions Figs. 1, bright soliton solution Fig. 2, exponential like function Fig. 3 and singular soliton solution Fig. 4. Moreover, the new types of soliton solutions realized by the (G’/G)-method appear in forms of hyperbolic function Fig. 5 and combination between bright soliton solution and dark soliton solution Fig. 6. To ensure the validity of the satisfied soliton solutions and prove the consistency, we will use the DTM method to extract the numerical solutions to the established soliton solutions achieved by these two proposed manners. Furthermore, the simulation for the 2-kind and 3-kind graphs shows the agreement of the analytical and numerical solutions in Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11). This agreement will be detected when we expand the soliton solutions achieved by these two manners using Taylor (McLaurin) and compared it with the DTM solutions. The soliton solutions of the CWK model and its identical numerical solutions will give new visions to the dynamical properties of the arising

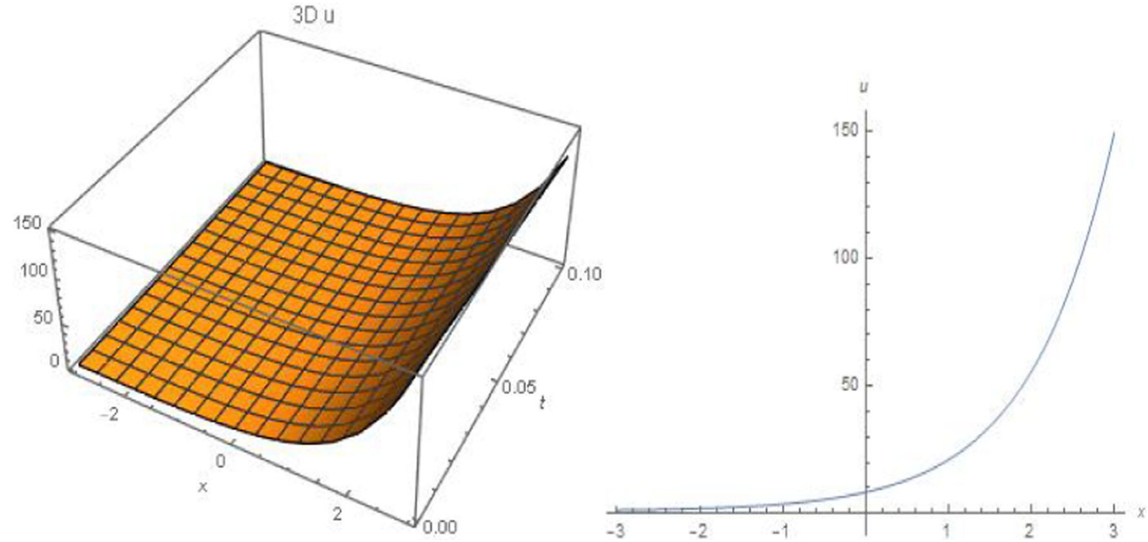

Fig. 3 This figure shows the solution of Eq. (1) given from Eq. (47) in 2D & 3D

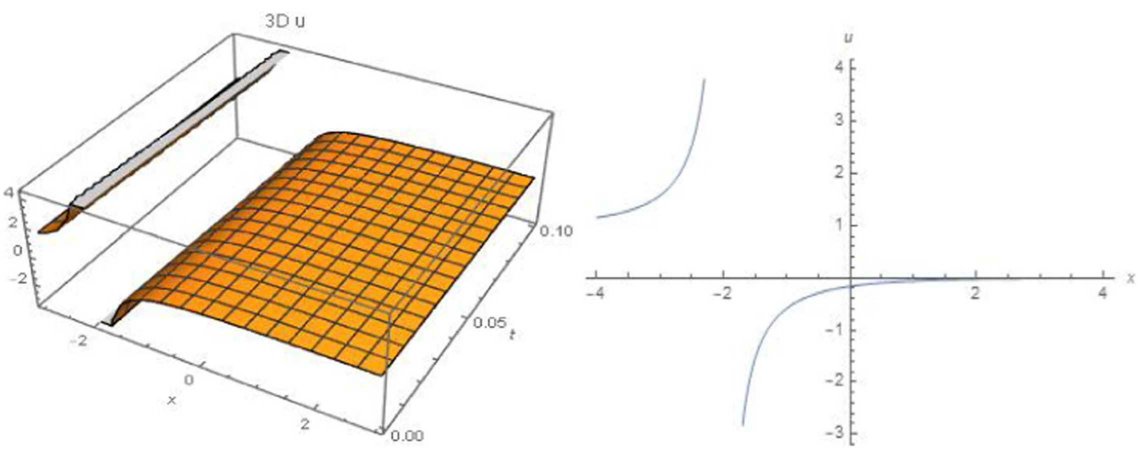

Fig. 4 This figure shows other form of solution for Eq. (1) given from Eq. (51)

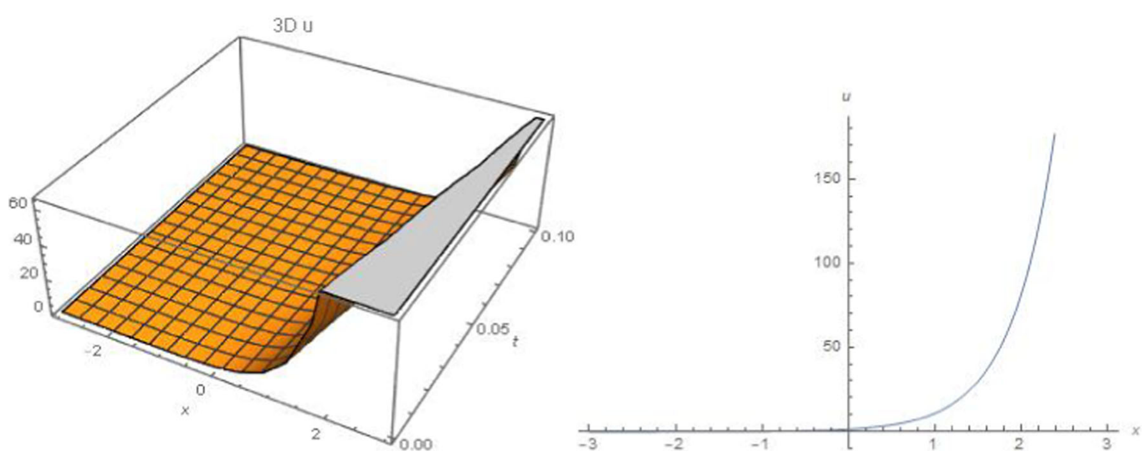

Fig. 5 This figure shows other forms of solution for the suggested model Eq. (1) given by Eq. (65)

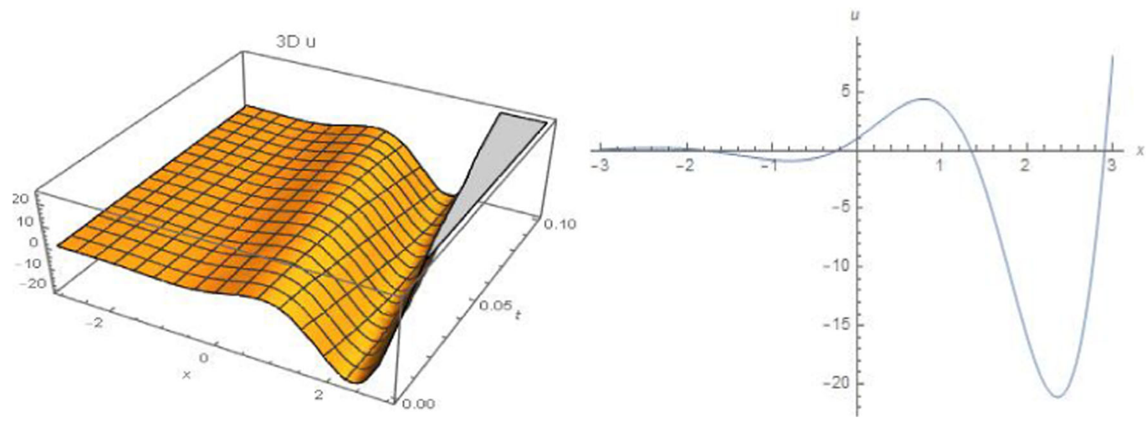

Fig. 6 This figure shows other forms of solution for proposed model Eq. (1) given by Eq. (69)

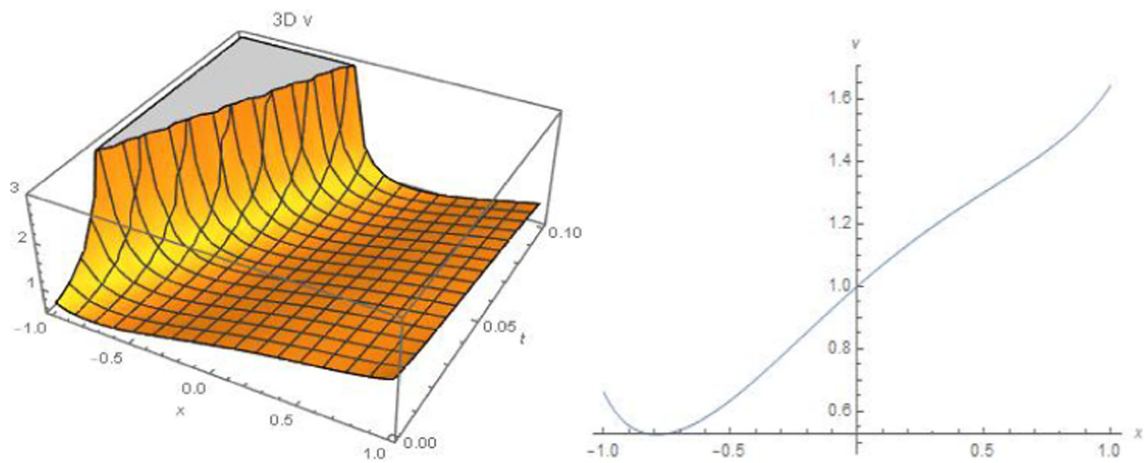

Fig. 7 DTM solution for proposed model Eq. (1) given by Eq. (79)

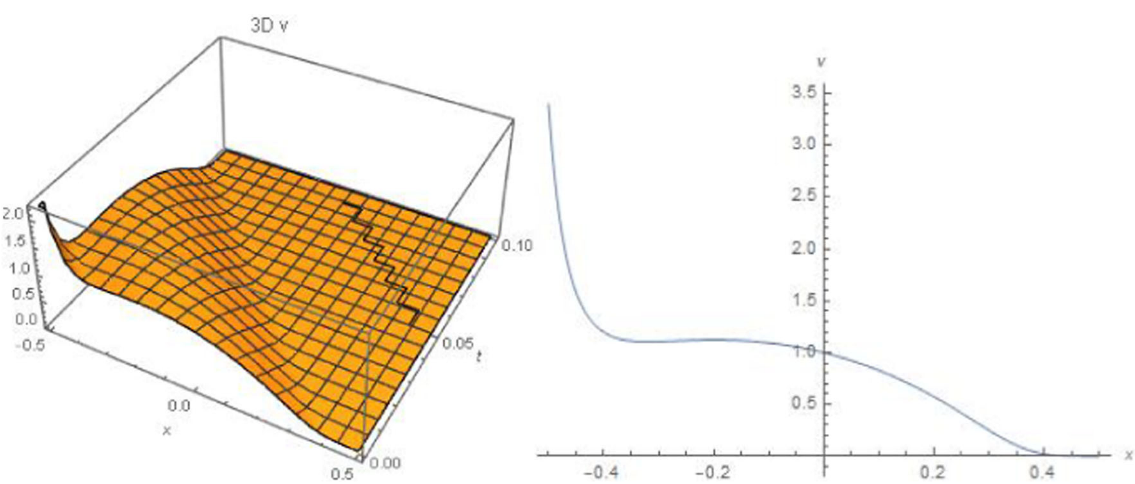

Fig. 8 DTM solution for proposed model Eq. (1) given by Eq. (83)

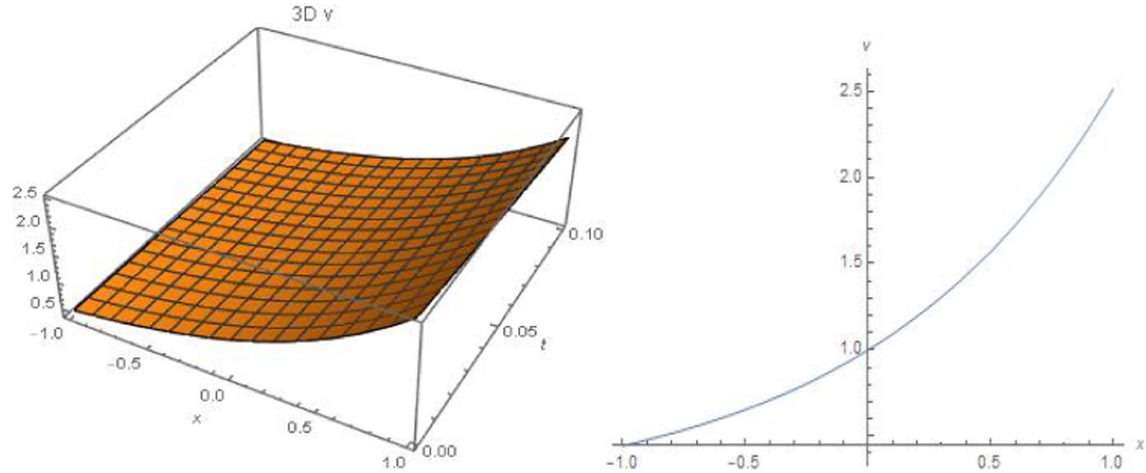

Fig. 9 DTM solution for suggested model Eq. (1) given by Eq. (91)

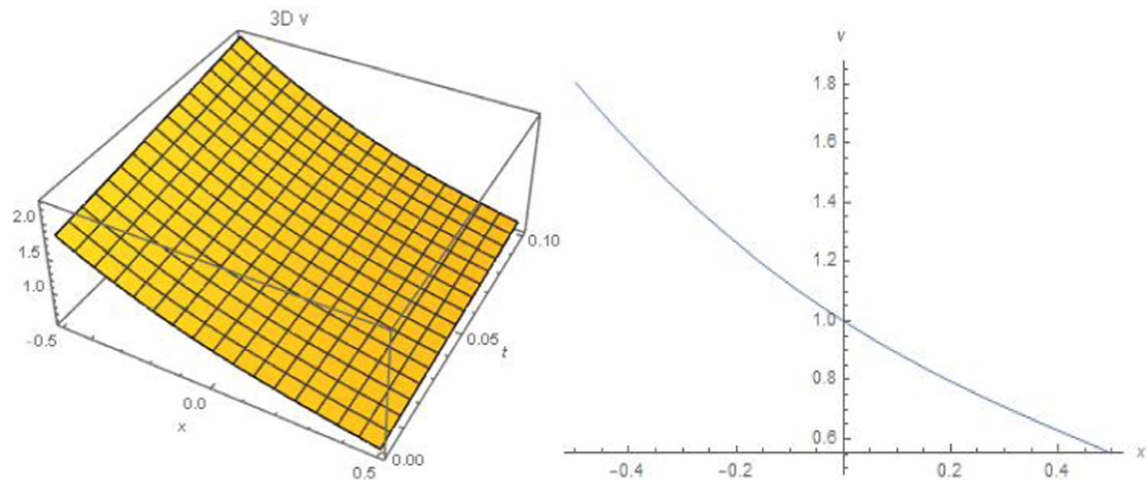

Fig. 10 DTM solution for our model Eq. (1) given by Eq. (95)

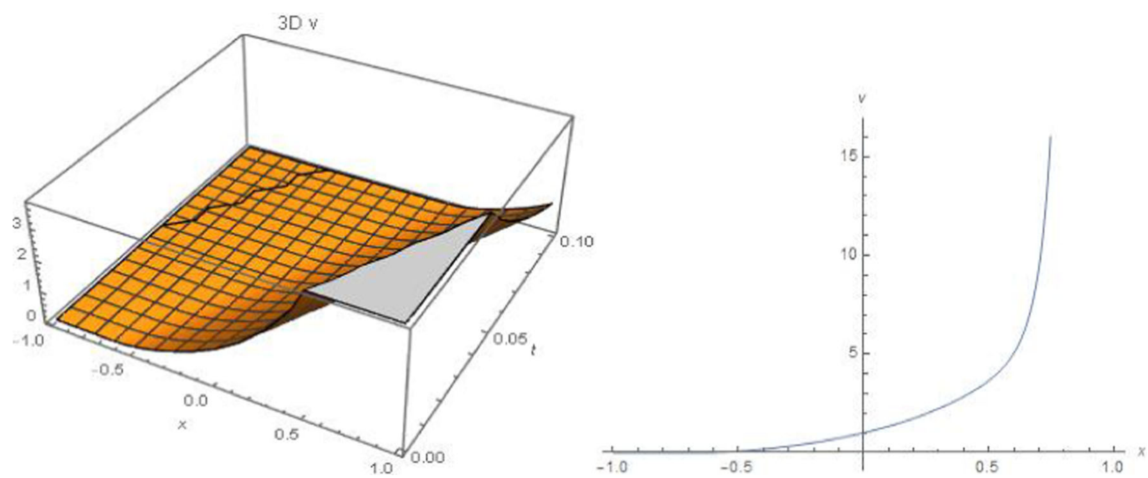

Fig. 11 DTM solution for our model Eq. (1) given by Eq. (103)

solitons that describe bacteria movement toward the light. The novelty of our satisfied solutions will be appeared when we execute the comparison between our realized solutions with that realized by [10, 11].

Author contributions All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Data Availability Statement The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

1. D. Levy, T. Requeijo, Modeling group dynamics of phototaxis: From particle systems to PDEs. Discret. Contin. Dyn. Syst.-B **9**, 103–128 (2008)
2. D. Levy, T. Requeijo, Stochastic models for phototaxis. Bull. Math. Biol. **70**, 1684–1706 (2008)
3. S. Ha, D. Levy, Particle, kinetic and fluid models for phototaxis. Discret. Contin. Dyn. Syst.-B **12**, 77–108 (2009)
4. A. Galante, S. Wisen, D. Bhaya, D. Levy, Modeling local interactions during the motion of cyanobacteria. J. Theor. Biol. **309**, 147–158 (2012)
5. A. Galante, D. Levy, Modeling selective local interactions with memory. Phys. D Nonlinear Phenom. **260**, 176–190 (2013)
6. D. Weinberg, D. Levy, Modeling selective local interactions with memory: Motion on a 2d lattice. Phys. D Nonlinear Phenom. **278–279**, 13–30 (2014)
7. K. Drescher, R. Goldstein, I. Tuval, Fidelity of adaptive phototaxis. Proc. Natl. Acad. Sci. U.S.A. **107**, 11171–11176 (2010)
8. P. Chavy-Waddy, T. Kolokolnikov, A local PDE model of aggregation formation in bacterial colonies. Nonlinearity **29**, 3174 (2016)
9. A.J. Bernoff, C.M. Topaz, Biological aggregation driven by social and environmental factors: A nonlocal model and its degenerate Cahn-Hilliard approximation. SIAM J. Appl. Dyn. Syst. **15**, 1528–1562 (2016)
10. L.R. Alejandro, G.G. Oswaldo, C.A. Guillermo, Analytical solutions to the Chavy-Waddy–Kolokolnikov model of bacterial aggregates in phototaxis by three integration schemes. Mathematics **11**, 2352 (2023)
11. N.A. Kudryashov, A.A. Kutukov, S.F. Lavrova, Properties of the generalized Chavy-Waddy–Kolokolnikov model for description of bacterial colonies. Commun. Nonlinear Sci. Numer. Simul., 2023, 107645
12. D. Lee, J. Huh, D. Jeong, J. Shin, A. Yun, J. Kim, Physical, mathematical, and numerical derivations of the Cahn-Hilliard equation. Comput. Mater. Sci. **81**, 216–225 (2014)
13. J.D. Murray, Mathematical Biology I. An Introduction, Volume 17 of Interdisciplinary Applied Mathematics; Springer: Berlin/Heidelberg, Germany (2002)
14. J.D. Murray, Mathematical Biology II: Spatial Models and Biomedical Applications, Volume 18 of Interdisciplinary Applied Mathematics; Springer: Berlin/Heidelberg, Germany (2003)
15. M. Kaplan, A. Akbulut, Application of two different algorithms to the approximate long water wave equation with conformable fractional derivative. Arab J. Basic Appl. Sci. **25**, 77–84 (2018)
16. X. Wang, X.G. Yue, M.K.A. Kaabar, A. Akbulut, M. Kaplan, A unique computational investigation of the exact traveling wave solutions for the fractional-order Kaup-Boussinesq and generalized Hirota Satsuma coupled KdV systems arising from water waves and interaction of long waves. J. Ocean Eng. Sci. (in press)
17. A.H. Arnous, M. Mirzazadeh, A. Akbulut, L. Akinyemi, Optical solutions and conservation laws of the Chen–Lee–Liu equation with Kudryashov's refractive index via two integrable techniques. Waves Random Complex Media (2022)
18. A. Akbulut, F. Taşcan, Lie symmetries, symmetry reductions and conservation laws of time fractional modified Korteweg–de Vries (mkdv) equation. Chaos Solitons Fractals **100**, 1–6 (2017)
19. E.H.M. Zahran, A. Bekir, R.A. Ibrahim, New optical soliton solutions of the popularized anti-cubic nonlinear Schrödinger equation versus its numerical treatment. Opt. Quant. Electron. **55**, 377 (2023)
20. E.H.M. Zahran, R.A. Ibrahim, D.U. Ozsahin, H. Ahmad, M.S.M. Shehata, New diverse exact optical solutions of the three dimensional Zakharov-Kuznetsov equation. Opt. Quant. Electron. **55**, 817 (2023)
21. A. Bekir, E.H.M. Zahran, Three distinct and impressive visions for the soliton solutions to the higher-order nonlinear Schrodinger equation. Optik-Int. J. Light Electron Opt. **228**, 166157 (2021)
22. A. Bekir, E.H.M. Zahran, Bright and dark soliton solutions for the complex Kundu-Eckhaus equation. Optik; Int. J. Light Electron Opt. **223**, 165233 (2020)
23. A. Bekir, E.M.H. Zahran, M.S.M. Shehata, Comparison between the new exact and numerical solutions of the Mikhailov-Novikov-Wang equation, Numerical Method of Partial Differential Equation, **40**, e22775 (2024)
24. E.H.M. Zahran, A. Bekir, R.A. Ibrahim, New impressive analytical optical soliton solutions to the Schrödinger-Poisson dynamical system against its numerical solutions. Opt. Quant. Electron. **55**, 212 (2023)
25. J. Zhou, *Differential transformation and its applications for electrical circuits* (Borneo Huazhong University Press, Wuhan, China, 2010)
26. F. Ziyaee, A. Tari, Differential transform method for solving two-dimensional Fredholm integral equation. Appl. Appl. Math. J. **10**(2), 852–863 (2015)
27. R.A. Ibrahim, S.A. Mohamed, Application of differential transform method with adomian polynomial for solving RLC circuits problems and higher order differential equations. Eng. Res. J. **51**(4), 89–95 (2022)
28. M. Kajani, N. Shehni, Differential transform method: an effective tool for solving nonlinear Volterra integro-differential equations. Aust. J. Basic Appl. Sci. **5**(9), 30–39 (2011)
29. F. Karakoç, H. Bereketoglu, Solutions of delay differential equations by using differential transform method. Int. J. Comput. Math. **86**(5), 914–923 (2009)